Unit 14

RATIO AND PROPORTION

14.1 Ratio and Proportion

We defined ratio $a:b=\frac{a}{b}$ as the comparison of two alike quantities a and b, called the elements (terms) of a ratio. (Elements must be expressed in the same units). Equality of two ratios was defined as proportion.

That is, if $a:b=c:d$, then a,b,c and d are said to be in proportion.

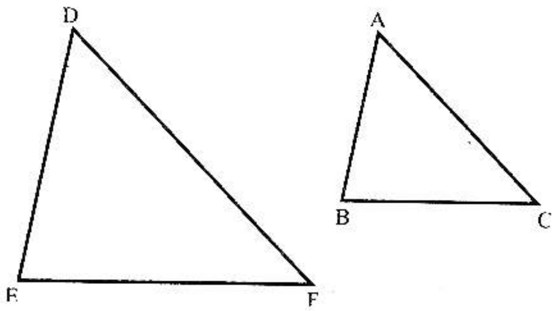
Similar Triangles

Equally important are the similar shapes. In particular the similar triangles that have many practical applications. For example, we know that a photographer can develop prints to different sizes from the same negative. In spite of the difference in size, these pictures look like each other. One photograph is simply an enlargement of another. They are said to be similar in shape. Geometrical figures can also be similar. e.g., if

In $\Delta ABC \longleftrightarrow \Delta DEF$

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$,

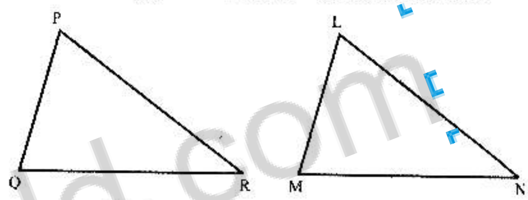
and $\frac{\overline{AB}}{\overline{DE}}=\frac{\overline{BC}}{\overline{EF}}=\frac{\overline{CA}}{\overline{FD}}$



then ΔABC and ΔDEF are called similar triangles which is symbolically written as $\Delta ABC \sim \Delta DEF$

It means that corresponding angles of similar triangles are equal and measures of their corresponding sides are proportional.

$\Delta PQR \cong \Delta LMN$ means that in



$\Delta PQR \longleftrightarrow \Delta LMN$

$\angle P \cong \angle L$, $\angle Q \cong \angle M$

$\angle R \cong \angle N$, $\overline{PQ} \cong \overline{LM}$,

$\overline{QR} \cong \overline{MN}$, $\overline{RP} \cong \overline{NL}$,

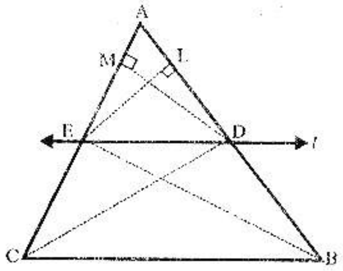
Now as $\frac{\overline{PQ}}{\overline{LM}}=\frac{\overline{QR}}{\overline{MN}}=\frac{\overline{RP}}{\overline{NL}}=1$

$\therefore$ $\Delta PQR \sim \Delta LMN$

Note:

Two congruent triangles are similar also. But two similar triangles are not necessarily congruent, as congruence of their corresponding sides is not necessary.

Theorem A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.



Given In ΔABC, the line l is intersecting the sides $\overline{AC}$ and $\overline{AB}$ at points E and D respectively such that $\overline{ED} \parallel \overline{CB}$.

To Prove

$m\overline{AD} : m\overline{BD} = m\overline{AE} : m\overline{EC}$

Construction

Join B to E and C to D. From D draw $\overline{DM} \perp \overline{AC}$ and from E draw $\overline{EL} \perp \overline{AB}$.

Proof

Statements	Reasons
In triangles BED and AED, $\overline{EL}$ is the common perpendicular.	
$\therefore\ \Delta BED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL}$ ....(i) and $\Delta AED = \frac{1}{2} \times m\overline{AD} \times m\overline{EL}$ ....(ii)	Area of a $\Delta = \frac{1}{2}$(base) (height)
Thus $\frac{\Delta BED}{\Delta AED} = \frac{m\overline{BD}}{m\overline{AD}}$ ....(iii)	Dividing (i) by (ii)
Similarly $\frac{\Delta CDE}{\Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}}$ ....(iv)	
But $\Delta BED \cong \Delta CDE$	Areas of triangles with common base and same altitudes are equal. Given that $\overline{ED} \parallel \overline{CB}$ so altitudes are equal.
$\therefore$ From (iii) and (iv), we have $\frac{m\overline{BD}}{m\overline{AD}} = \frac{m\overline{EC}}{m\overline{AE}}$ or $\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$	Taking reciprocal of both sides.
Hence $m\overline{AD} : m\overline{BD} = m\overline{AE} : m\overline{EC}$	

Note:

From the above theorem we also have

$$\frac{m\overline{BD}}{m\overline{AB}} = \frac{m\overline{CE}}{m\overline{AC}} \text{ and } \frac{m\overline{AD}}{m\overline{AB}} = \frac{m\overline{AE}}{m\overline{AC}}$$

Corollaries

a) If $\frac{m\overline{AD}}{m\overline{AB}} = \frac{m\overline{AE}}{m\overline{AC}}$, then $\overline{DE} \parallel \overline{BC}$

b) If $\frac{m\overline{AB}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{EC}}$, then $\overline{DE} \parallel \overline{BC}$

Note:

i) Two points determine a line and three non-collinear points determine a plane.

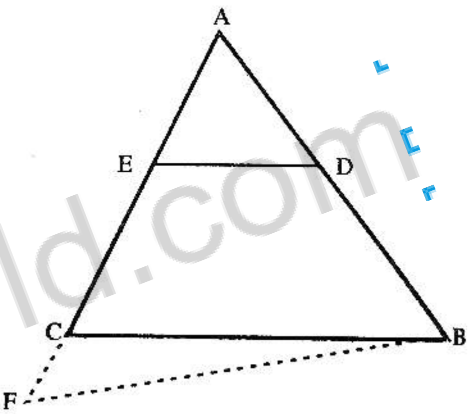
ii) A line segment has exactly one midpoint.

iii) If two intersecting lines from equal adjacent angles, the lines are perpendicular.

Theorem

(Converse of Theorem)

If a line segment intersects the two sides of a triangle in the same ratio then it is parallel to the third side.



Given In ΔABC, $\overline{ED}$ intersects $\overline{AB}$ and $\overline{AC}$ such that $m\overline{AD} : m\overline{BD} = m\overline{AE} : m\overline{EC}$

To Prove $\overline{ED} \parallel \overline{CB}$

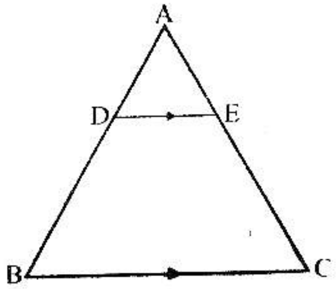
Construction If $\overline{ED} \nparallel \overline{CB}$, then draw $\overline{BF} \parallel \overline{DE}$ to meet $\overline{AC}$ produced at F.

Proof

Statements	Reasons
In ΔABF	
$\overline{DE} \parallel \overline{BF}$	Construction
$\therefore \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EF}}$ ....(i)	(A line parallel to one side of a triangle divides the other two sides proportionally)
But $\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$ ....(ii)	Given
$\therefore \frac{m\overline{AE}}{m\overline{EF}} = \frac{m\overline{AE}}{m\overline{EC}}$	From (i) and (ii)
or $m\overline{EF} = \overline{EC}$	
which is possible only if point F is coincident with C.	(Property of real numbers)
$\therefore$ Our supposition is wrong.	
Hence $\overline{ED} \parallel \overline{CB}$	

Exercise 14.1

1. In ΔABC, $\overline{DE} \parallel \overline{BC}$



i) $\overline{AD} = 1.5\text{ cm}, \overline{BD} = 3\text{ cm}$,
$\overline{AE} = 1.3\text{cm}$ then find $\overline{CE}$.

ii) If $\overline{AD} = 2.4\text{ cm}$, $\overline{AE} = 3.2\text{ cm}$, $\overline{EC} = 4.8\text{ cm}$, find $\overline{AB}$

iii) If $\dfrac{\overline{AD}}{\overline{DB}} = \dfrac{3}{5}, \overline{AC} = 4.8\text{ cm}$, find $\overline{AE}$

iv) If $\overline{AD} = 2.4\text{ cm}$, $\overline{AE} = 3.2\text{ cm}$, $\overline{DE} = 2\text{ cm}$, $\overline{BC} = 5\text{ cm}$, find $\overline{AB}, \overline{DB}, \overline{AC}, \overline{CE}$

v) If $\overline{AD} = 4x-3, \overline{AE} = 8x-7$, $\overline{BD} = 3x-1$, and $\overline{CE} = 5x-3$, find the value of x

In ΔABC, $\overline{DE} \parallel \overline{BC}$

(i)

$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$$

$$\frac{1.5}{3} = \frac{1.3}{m\overline{EC}}$$

$$m\overline{EC} = \frac{3 \times 1.3}{1.5}$$

$$= 2.6\text{cm}$$

(ii) In ΔABC, $\overline{DE} \parallel \overline{BC}$

$$m\overline{AB} = m\overline{AD} + m\overline{BD}$$

Let $m\overline{DB} = x\text{ cm}$

Now $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$

$$\frac{2.4}{x} = \frac{3.2}{4.8}$$

$$x = \frac{4.8 \times 2.4}{3.2}$$

$$x = \frac{48 \times 24}{10 \times 32}$$

$$x = 3.6\text{cm}.$$

$\therefore$ $m\overline{AB} = m\overline{AD} + m\overline{BD}$

$$m\overline{AB} = 2.4 + 3.6 = 6\text{cm}$$

(iii) $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{3}{5}, m\overline{AC} = 4.8\text{cm}$

In $\Delta ABC, \overline{DE} \parallel \overline{BC}$

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AC} - m\overline{CE}}{m\overline{CE}}$$

$$\frac{3}{5} = \frac{4.8 - m\overline{CE}}{m\overline{CE}}$$

$$3m\overline{CE} = 5(4.8 - m\overline{CE})$$

$$3m\overline{CE} = 24 - 5m\overline{CE}$$

$$3m\overline{CE} + 5m\overline{CE} = 24$$

$$8m\overline{CE} = 24$$

$$m\overline{CE} = \frac{24}{8} = 3\text{cm}$$

$$m\overline{AE} = m\overline{AC} - m\overline{CE}$$

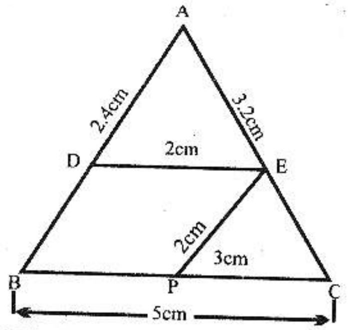
$$= 4.8 - 3$$

$$m\overline{AE} = 1.8\text{cm}$$

(iv) $m\overline{AD} = 2.4cm$,

$m\overline{AE} = 3.2\,cm, m\overline{DE} = 2cm,\ m\overline{BC} = 5cm.$

$m\overline{AB} = ?\ m\overline{DB} = ?\ m\overline{AC} = ?\ m\overline{CE} = ?$



$\overline{EP} \parallel \overline{AB}$

DEPB is a parallelogram, then

$m\overline{PB} = mDE = 2cm.$

$m\overline{CP} = 5 - 2 = 3cm$

In $\Delta ABC,\ \overline{EP} \parallel \overline{AB}$

$$\frac{m\overline{CE}}{m\overline{EA}} = \frac{m\overline{CP}}{m\overline{PB}}$$

$$\frac{m\overline{CE}}{3.2} = \frac{3}{2}$$

$$m\overline{CE} = \frac{3 \times 3.2}{2}$$

$$m\overline{CE} = 3 \times 1.6 = 4.8cm$$

Now $\overline{DE} \parallel \overline{BC}$, in ΔABC

$$\frac{m\overline{BD}}{m\overline{AD}} = \frac{m\overline{CE}}{m\overline{AE}}$$

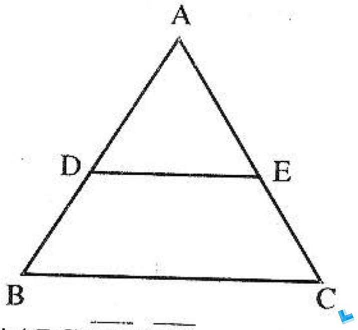
$$\frac{m\overline{BD}}{2.4} = \frac{4.8}{3.2}$$

$$m\overline{BD} = \frac{2.4 \times 4.8}{3.2} = 3.6cm$$

$m\overline{AB} = m\overline{AD} + m\overline{DB}$
$= 2.4 + 3.6$
$= 6.0cm$

$m\overline{AC} = m\overline{AE} + m\overline{EC}$
$= 3.2 + 4.8$
$= 8.0cm.$

(v) If $\overline{AD} = 4x - 3, \overline{AE} = 8x - 7, \overline{BD} = 3x - 1$ and $\overline{CE} = 5x - 3$, Find the value of x



In $\Delta ABC, \overline{DE} \parallel \overline{BC}$

$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{CE}}$$

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$(4x-3)(5x-3) = (8x-7)(3x-1)$$

$$20x^2 - 27x + 9 = 24x^2 - 29x + 7$$

$$20x^2 - 24x^2 - 27x + 29x + 9 - 7 = 0$$

$$-4x^2 + 2x + 2 = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$2x(x-1) + 1(x-1) = 0$$

$$(x-1)(2x+1) = 0$$

$x - 1 = 0$ or $2x + 1 = 0$

$x = 1$ or $2x = -1$

$x = 1$ or $x = \frac{-1}{2}$

But $x = \frac{-1}{2}$ not possible

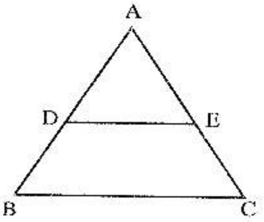
So $x = 1$

2. If ΔABC is an isosceles triangle, $\angle A$ is vertex angle and $\overrightarrow{DE}$ intersects the

sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that.

$$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$$

Prove that ΔADE is also an isosceles triangle.



In ΔABC, $\angle A$ is vertical angle and $\overline{AB} \cong \overline{AC}$

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

$$\frac{m\overline{DB}}{m\overline{AD}} = \frac{m\overline{EC}}{m\overline{AE}}$$

$$\frac{m\overline{DB} + m\overline{AD}}{m\overline{AD}} = \frac{m\overline{EC} + m\overline{AE}}{m\overline{AE}}$$

$$\frac{m\overline{AB}}{m\overline{AD}} = \frac{m\overline{AC}}{m\overline{AE}}$$

Now $\quad m\overline{AB} = m\overline{AC}$

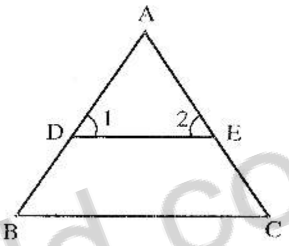
$$m\overline{AD} = m\overline{AE}$$

Δ ADE is an isosceles triangle.

3. In an equilateral triangle ABC shown in the figure.

$$m\overline{AE} : m\overline{AC} = m\overline{AD} : m\overline{AB}$$

Find all three angles of ΔADE and name it also.



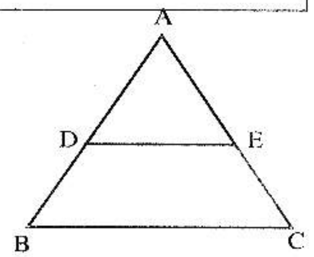
Given: ΔABC is an equilateral triangle.

$$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$$

To Prove: Find all angles of ΔADE

Statements	Reasons
$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$	Given
Then $\overline{DE} \parallel \overline{BC}$	Proved
ΔABC is equilateral triangle	
Then $m\angle A = m\angle B = m\angle C = 60^\circ$	Corresponding angle
$\overline{DE} \parallel \overline{BC}$	
$m\angle 1 = m\angle B = 60^\circ$	
$m\angle 2 = m\angle C = 60^\circ$	
$m\angle A = 60^\circ$	

4. Prove that the line segment drawn through the mid-point of one side of a triangle and parallel to another side bisects the third side.



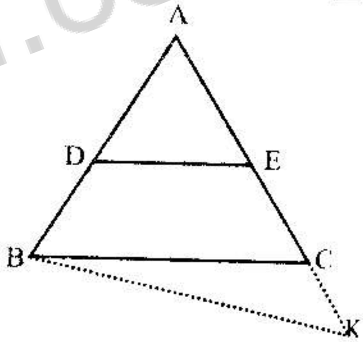
Given in $\Delta ABC, \overline{DE}$ is such that $m\overline{AD} = m\overline{DB}$ and $\overline{DE} \parallel \overline{BC}$

To Prove:

$m\overline{AE} = m\overline{EC}$

Statements	Reasons
In ΔABC	
$\overline{DE} \parallel \overline{BC}$	Given
$\dfrac{m\overline{AD}}{m\overline{BD}} = \dfrac{m\overline{AE}}{m\overline{EC}}$ .....(i)	
$m\overline{AD} = m\overline{DB}$	Given
$\dfrac{m\overline{DB}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$	Put $m\overline{AD} = m\overline{DB}$ in (i)
$1 = \dfrac{m\overline{AE}}{m\overline{EC}}$	
$m\overline{AE} = m\overline{EC}$	

5. Prove that the line segment joining the mid-points of any two sides of a triangle is parallel to the third side.



Given:

In ΔABC, points D, E are such that $m\overline{AD} = m\overline{DB}$

$m\overline{AE} = m\overline{EC}$

$\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$

To Prove:

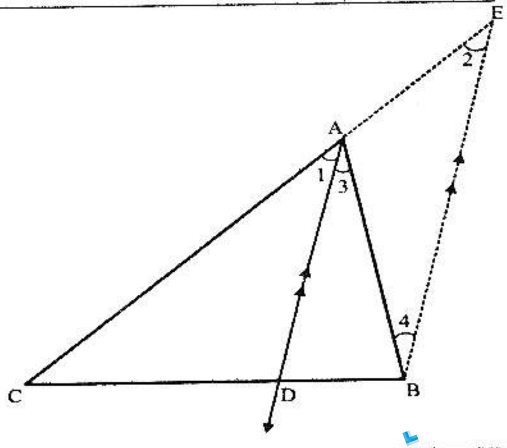
$\overline{DE} \parallel \overline{BC}$

Statements	Reasons
If $\overline{DE} \nparallel \overline{BC}$	
Then suppose $\overline{DE} \parallel \overline{BK}$	
Now $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EK}}$ .....(i)	
$\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$ .....(ii)	Given
$\dfrac{m\overline{AE}}{m\overline{EK}} = \dfrac{m\overline{AE}}{m\overline{EC}}$	From (i) and (ii)
$m\overline{EK} = m\overline{EC}$	

It is possible only when point K lies on the point C. Thus $\overline{DE} \parallel \overline{BC}$	

Theorem

The internal bisector of an angle of a triangle divides the side opposite to it in the ratio of the lengths of the sides containing the angle.



Given: In ΔABC internal angle bisector of $\angle A$ meets $\overline{CB}$ at the point D.

To Prove: $m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$

Construction:

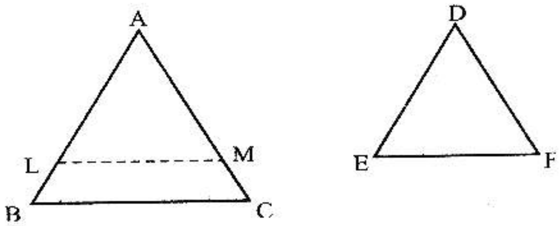
Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet $\overline{CA}$ produced at E.

Proof:

Statements	Reasons
$\because$ $\overline{AD} \parallel \overline{EB}$ and $\overline{EC}$ intersects them,	Construction
$\therefore$ $m\angle 1 = m\angle 2$ ....(i)	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$	
and $\overline{AB}$ intersects them,	
$\therefore$ $m\angle 3 = m\angle 4$ ....(ii)	Alternate angles
But $m\angle 1 = m\angle 3$	Given
$\therefore$ $m\angle 2 = m\angle 4$	From (i) and (ii)
and $\overline{AB} \cong \overline{AE}$ or $\overline{AE} \cong \overline{AB}$	In a Δ, the sides opposite to congruent angles are also congruent.
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore$ $\dfrac{m\overline{BD}}{m\overline{DC}} = \dfrac{m\overline{EA}}{m\overline{AC}}$	By Theorem
or $\dfrac{m\overline{BD}}{m\overline{DC}} = \dfrac{m\overline{AB}}{m\overline{AC}}$	$m\overline{EA} = m\overline{AB}$ (proved)
Thus $m\overline{BD} : m\overline{DC} = m\overline{AB} : \overline{AC}$	

Theorem: If two triangles are similar, then the measures of their corresponding sides are proportional.

Given: $\Delta ABC \sim \Delta DEF$



i.e., $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$

To Prove:

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

Construction:

i) Suppose that $m\overline{AB} > m\overline{DE}$

ii) $m\overline{AB} \leq m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$. Join L and M by the line segment LM.

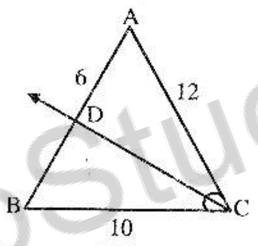
Proof:

Statements	Reasons
i) In $\Delta ALM \longleftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S. Postulate
and $\angle L \cong \angle E$, $\angle M \cong \angle F$,	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$, and $\angle F \cong \angle C$	Given
$\therefore$ $\angle L \cong \angle B$, $\angle M \cong \angle C$,	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal.
Hence $\frac{m\overline{AL}}{m\overline{AB}} = \frac{m\overline{AM}}{m\overline{AC}}$	By Theorem
or $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}}$ .....(i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (construction)
Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{EF}}{m\overline{BC}}$ .....(ii)	
Thus $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}} = \frac{m\overline{EF}}{m\overline{BC}}$	by (i) and (ii)
or $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$	by taking reciprocals
ii) If $m\overline{AB} < m\overline{DE}$, it can similarly be	

proved by taking intercepts on the sides of ΔDEF	
If $m\overline{AB} = m\overline{DE}$,	
then in $\Delta ABC \longleftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\angle B \cong \angle E$	Given
and $\overline{AB} \cong \overline{DE}$	
so $\Delta ABC \cong \Delta DEF$	$A.S.A \cong A.S.A$
Thus $\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}} = 1$	$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
Hence the result is true for all the cases.	

Exercise 14.2

1. In ΔABC as shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D, $m\overline{BD}$ is equal to a) 5 b) 16 c) 10 d) 18

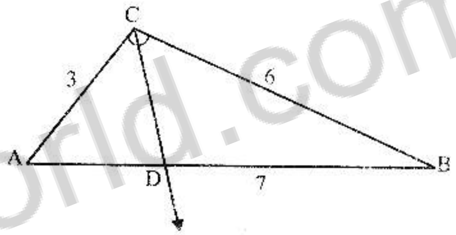


Ans. $\dfrac{m\overline{BD}}{m\overline{DA}} = \dfrac{m\overline{BC}}{m\overline{CA}}$

$\dfrac{m\overline{BD}}{6} = \dfrac{10}{12}$

$m\overline{BD} = \dfrac{10}{12} \times 6 = 5$

2. In ΔABC as shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$. If $m\overline{AC} = 3$, $m\overline{CB} = 6$ and $m\overline{AB} = 7$, then find $m\overline{AD}$ and $m\overline{DB}$.



Ans. $m\overline{AD} = x$

$m\overline{BD} = 7 - x$

$\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AC}}{m\overline{CB}}$

$\dfrac{x}{7-x} = \dfrac{3}{6}$

$\dfrac{x}{7-x} = \dfrac{1}{2}$

$2x = 1(7-x)$

$2x = 7 - x$

$3x = 7 \Rightarrow \quad x = \dfrac{7}{3}$

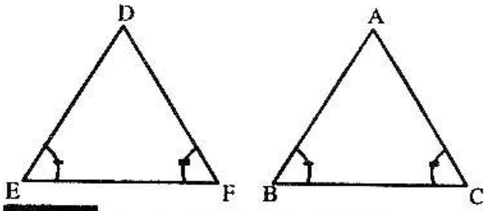
$m\overline{AD} = \dfrac{7}{3}$

$m\overline{DB} = 7 - x$

$$= 7 - \frac{7}{3}$$

$$= \frac{21-7}{3} = \frac{14}{3}$$

3. Show that in any correspondence of two triangles if two angles of one triangle are congruent to the corresponding angles of the other, then the triangles are similar.



Given: In ΔABC and ΔDEF

$m\angle B = m\angle E$

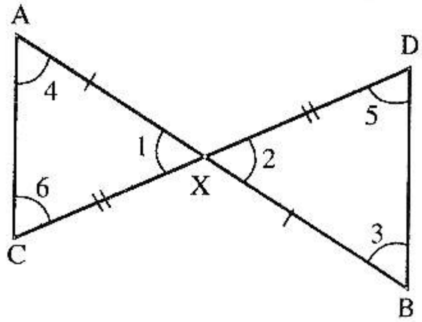
$m\angle C = m\angle F$

To Prove: $\Delta ABC \sim \Delta DEF$

Proof:

Statements	Reasons
$m\angle B + m\angle C + m\angle A = 180^\circ$ ---(i)	Sum of interior angles of triangle is 180°
$m\angle E + m\angle F + m\angle D = 180^\circ$....(ii)	Given
$m\angle B + m\angle C + m\angle D = 180^\circ$...(iii) $m\angle A - m\angle D = 0$ $m\angle A = m\angle D$ All Angles of ΔDEF and ΔABC are congruent Thus $\Delta ABC \sim \Delta DEF$.	Subtracting (i) from (ii)

4. If line segments $\overline{AB}$ and are $\overline{CD}$ intersecting at point X and $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$ then show that ΔAXC and ΔBXD are similar.



Given:

$\overline{AB}$ and $\overline{CD}$ intersect each other at point x and

$$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$$

To Prove:

$\Delta AXC \sim \Delta BXD$

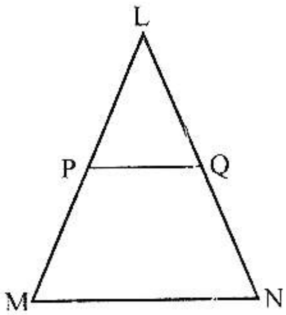
Proof:

Statements	Reasons
In ΔAXC and ΔBXD	
$\angle 1 \cong \angle 2$	Vertical angles
$\dfrac{m\overline{AX}}{m\overline{XB}} = \dfrac{m\overline{CX}}{m\overline{XD}}$	Given
Then $\overline{AC} \parallel \overline{BD}$	
$\angle 4 \cong \angle 3$	Alternate angles
$\angle 6 \cong \angle 5$	
Thus $\dfrac{m\overline{AX}}{m\overline{XB}} = \dfrac{m\overline{CX}}{m\overline{XD}} = \dfrac{m\overline{AC}}{m\overline{DB}}$	
Hence ΔAXC and ΔBXD are similar.	

5. Which of the following are true and which are false?

i.	Congruent triangles are of same size and shape.	True
ii.	Similar triangles are of same shape but different sizes.	True
iii.	Symbol used for congruent is '~'.	False
iv.	Symbol used for similarity is '≅'.	False
v.	Congruent triangles are similar.	True
vi.	Similar triangles are congruent.	False
vii.	A line segment has only one mid point.	True
viii.	One and only one line can be drawn through two points.	True
ix.	Proportion is non-equality of two ratios.	False
x.	Ratio has no unit.	True

6. In ΔLMN show in the figure, $\overline{MN} \parallel \overline{PQ}$.



i) If $m\overline{LM} = 5$cm, $m\overline{LP} = 2.5$cm, $m\overline{LQ} = 2.3$cm, then find $m\overline{LN}$.

ii) If $m\overline{LM} = 6$cm, $m\overline{LQ} = 2.5cm$, $m\overline{QN} = 5$cm, then find $m\overline{LP}$.

Given: In ΔLMN, $\overline{MN} \parallel \overline{PQ}$

$m\overline{LM} = 5$cm, $m\overline{LP} = 2.5$cm, $m\overline{LQ} = 2.3$cm

To Prove: mLN = ?

Proof:

Statements	Reasons
$\dfrac{m\overline{LN}}{m\overline{LQ}} = \dfrac{m\overline{LM}}{m\overline{LP}}$	$\overline{PQ} \parallel \overline{MN}$ (Given)

$\frac{m\overline{LN}}{2.3}=\frac{5}{2.5}$ $m\overline{LN}=\frac{5\times2.3}{2.5}$ $=\frac{5\times23}{25}$ $=4.6cm$	Putting Values

(ii)

Given: $\Delta LMN, \overline{MN}\parallel\overline{PQ}$

$m\overline{QN}=5cm, m\overline{LQ}=2.5cm, m\overline{LM}=6cm.$

To prove: $m\overline{LP}=?$

Proof:

$$\frac{m\overline{LP}}{m\overline{LM}}=\frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{m\overline{LP}}{m\overline{LM}}=\frac{m\overline{LQ}}{m\overline{LQ}+m\overline{QN}}$$

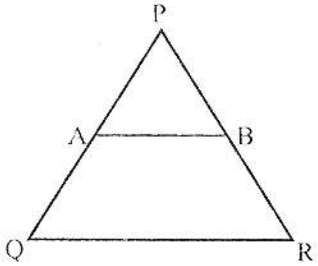
$$\frac{m\overline{LP}}{6}=\frac{2.5}{2.5+5}$$

$$m\overline{LP}=\frac{2.5}{7.5}\times6$$

$$m\overline{LP}=\frac{1}{3}\times6$$

$$=2cm.$$

7. In the shown figure, let $m\overline{PA}=8x-7$, $m\overline{PB}=4x-3$, $m\overline{AQ}=5x-3$, $m\overline{BR}=3x-1$. Find the value of x if $\overline{AB}\parallel\overline{QR}$.



If $\overline{AB}\parallel\overline{QR}$ then

$$\frac{m\overline{PA}}{m\overline{AQ}}=\frac{m\overline{PB}}{m\overline{BR}}$$

Putting values

$$\frac{8x-7}{5x-3}=\frac{4x-3}{3x-1}$$

$$(8x-7)(3x-1)=(5x-3)(4x-3)$$

$$24x^2-8x-21x+7=20x^2-15x-12x+9$$

$$24x^2-29x+7=20x^2-27x+9$$

$$24x^2-20x^2-29x+27x+7-9=0$$

$$4x^2-2x-2=0$$

$$2x^2-x-1=0$$

$$2x^2-2x+x-1=0$$

$$2x(x-1)+1(x-1)=0$$

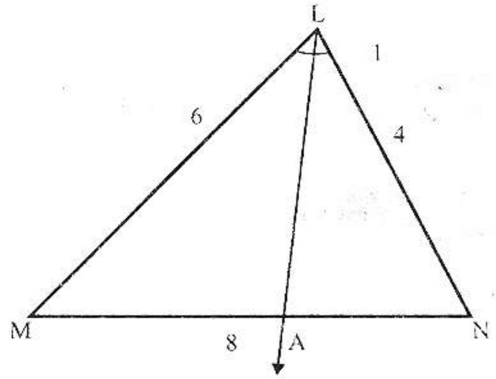
$$(2x+1)(x-1)=0$$

$$2x+1=0 \quad \text{or} \quad x-1=0$$

$$2x=-1 \qquad x=1$$

$$x=\frac{-1}{2}$$

8. In ΔLMN shown in the figure $\overline{LA}$ bisects $\angle L$. If $m\overline{LN}=4$, $m\overline{LM}=6$, $m\overline{MN}=8$, then find $m\overline{MA}$ and $m\overline{AN}$.



Given: In $\Delta LMN, \overline{LA}$ is angle bisector of $\angle L$.

$m\overline{LM} = 6cm, m\overline{LN} = 4cm, m\overline{MN} = 8cm.$

To Prove: $m\overline{MA} = ?$, $m\overline{AN} = ?$

Proof:

Let $m\overline{AN} = xcm$

$m\overline{MA} = 8 - xcm$

$$\frac{m\overline{MA}}{m\overline{AN}} = \frac{m\overline{LM}}{m\overline{LN}}$$

Putting values

$$\frac{8-x}{x} = \frac{6}{4}$$

$4(8-x) = 6x$

$32 - 4x = 6x$

$32 = 6x + 4x$

$10x = 32$

$x = \frac{32}{10} = 3.2$

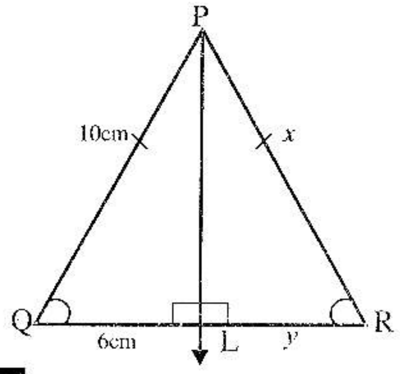
$\therefore$ $m\overline{AN} = 3.2cm.$

$m\overline{MA} = 8 - x$

$= 8 - 3.2$

$= 4.8cm.$

9. In Isosceles ΔPQR shown in the figure, find the value of x and y.



Given:

In $\Delta PQR, \overline{PQ} \cong \overline{PR}$ and $\overline{PL} \perp \overline{QR}$.

To Prove: $x = ?$ $y = ?$

Proof:

In ΔPRL and ΔPQL

$m\overline{PQ} = m\overline{PR}$ ... (i)	Isosceles triangle
$m\angle PLQ = m\angle PLR$	Each of right angle
$m\overline{PL} = m\overline{PL}$	Common
$\Delta PQL \cong \Delta PRL$	H.S. $\cong$ H.S

$m\overline{QL} = m\overline{LR}$

$6 = y$

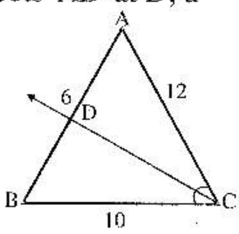
$\Rightarrow y = 6cm.$

From (i) $x = 10cm.$

OBJECTIVE

1. In ΔABC as shown in figure, $\overrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D, a $m\overline{BD}$ is equal to:

(a) 5
(b) 16
(c) 10
(d) 18



2. In ΔABC shown in figure, $\overrightarrow{CD}$ bisects $\angle C$, if $m\overline{AC} = 3$, $m\overline{CB} = 6$ and $m\overline{AB} = 7$ then

(i) $\overline{AD}$ = ____

(a) $\frac{7}{3}$ (b) $\frac{14}{3}$

(c) $\frac{9}{2}$ (d) $\frac{11}{2}$

(ii) $m\overline{BD}$ = ____

(a) $\frac{7}{3}$ (b) $\frac{14}{3}$

(c) $\frac{15}{2}$ (d) $\frac{11}{2}$

3. One and only one line can be drawn through ___ points:

(a) Two (b) Three

(c) Four (d) Five

4. The ratio between two alike quantities is defined as:

(a) a : b

(b) b : a

(c) a : b = c : d

(d) None

5. If a line segment intersects the two sides of a triangle in the same ratio then it is parallel to the __ side:

(a) Third (b) Fourth

(c) Second (d) None

6. Two triangles are said to be similar if these are equiangular and their corresponding sides are ___

(a) Proportional

(b) congruent

(c) concurrent

(d) None

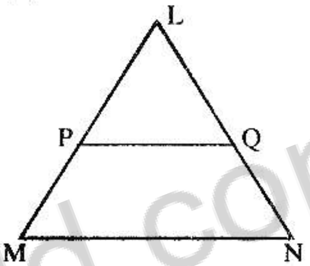
7. In ΔLMN shown in the figure $\overline{MN} \parallel \overline{PQ}$ if $m\overline{LM} = 5cm$, $m\overline{LP} = 2.5cm$, $m\overline{LQ} = 2.3cm$ then $m\overline{LN} =$ ___ :

(a) 4.6cm

(b) 4.5cm

(c) 3.5cm

(d) 4.0



ANSWER KEY

1.	a	**2.**	(i) a (ii) b	**3.**	a	**4.**	a	**5.**	a
6.	a	**7.**	a						